DPP - Daily Practice Problems

Name :

Date :

Start Time :

End Time :

PHYSICS 15

SYLLABUS : Rotational Motion – 1: Basic concepts of rotational motion, moment of a force, torque, angular momentum and its conservation with application

Max. Marks : 112 **Time : 60 min.**

GENERAL INSTRUCTIONS

- The Daily Practice Problem Sheet contains 28 MCQ's. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.
- You have to evaluate your Response Grids yourself with the help of solution booklet.
- Each correct answer will get you 4 marks and 1 mark shall be deduced for each incorrect answer. No mark will be given/ deducted if no bubble is filled. Keep a timer in front of you and stop immediately at the end of 60 min.
- The sheet follows a particular syllabus. Do not attempt the sheet before you have completed your preparation for that syllabus. Refer syllabus sheet in the starting of the book for the syllabus of all the DPP sheets.
- After completing the sheet check your answers with the solution booklet and complete the Result Grid. Finally spend time to analyse your performance and revise the areas which emerge out as weak in your evaluation.

DIRECTIONS (Q.1-Q.20) : There are 20 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which **ONLY ONE** choice is correct.

Q.1 A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Four objects each of mass m, are kept gently to the opposite ends of two perpendicular diameters of the ring. The angular velocity of the ring will be

(a) $\frac{M\omega}{M+4m}$ (b) $\frac{(M+4m)\omega}{M}$ (c) $\frac{(M-4m)\omega}{M+4m}$ (d) $\frac{M\omega}{4m}$

Q..2 The angular momentum of a system of particles is conserved

(a) When no external force acts upon the system

(b) When no external torque acts upon the system

(c) When no external impulse acts upon the system

(d) When axis of rotation remains same

Q.3 Two rigid bodies A and B rotate with rotational kinetic energies E_A and E_B respectively. The moments of inertia of A and B about the axis of rotation are I_A and I_B respectively. If $I_A = I_B/4$ and $E_A = 100\,E_B$, the ratio of angular momentum (L_A) of A to the angular momentum (L_B) of B is

(a) 25 (b) 5/4 (c) 5 (d) 1/4

Q.4 A uniform heavy disc is rotating at constant angular velocity ω about a vertical axis through its centre and perpendicular to the plane of the disc. Let L be its angular momentum. A lump of plasticine is dropped vertically on the disc and sticks to it. Which of the following will be constant?

(a) ω (b) ω and L both

(c) L only (d) Neither ω nor L

RESPONSE GRID	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ

Space for Rough Work

Q.5 Two discs of moment of inertia I_1 and I_2 and angular speeds ω_1 and ω_2 are rotating along collinear axes passing through their centre of mass and perpendicular to their plane. If the two are made to rotate combindly along the same axis the rotational KE of system will be

(a) $\frac{I_1\omega_1 + I_2\omega_2}{2(I_1+I_2)}$ (b) $\frac{(I_1+I_2)(\omega_1+\omega_2)^2}{2}$

(c) $\frac{(I_1\omega_1+I_2\omega_2)^2}{2(I_1+I_2)}$ (d) None of these

Q.6 A particle performs uniform circular motion with an angular momentum L. If the frequency of a particle's motion is doubled and its kinetic energy is halved, the angular momentum becomes.

(a) 2 L (b) 4 L (c) L/2 (d) L/4

Q.7 A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω about the same axis. The final angular velocity of the combination of discs is

(a) $\frac{I_2\omega}{I_1+I_2}$ (b) ω (c) $\frac{I_1\omega}{I_1+I_2}$ (d) $\frac{(I_1+I_2)\omega}{I_1}$

Q.8 Calculate the angular momentum of a body whose rotational energy is 10 joule. If the angular momentum vector coincides with the axis of rotation and its moment of inertia about this axis is 8×10^{-7} kg m^2

(a) 4×10^{-3} kg m^2/s (b) 2×10^{-3} kg m^2/s
(c) 6×10^{-3} kg m^2/s (d) None of these

Q.9 If the earth is treated as a sphere of radius R and mass M. Its angular momentum about the axis of rotation with period T is

(a) $\frac{\pi MR^3}{T}$ (b) $\frac{MR^2\pi}{T}$ (c) $\frac{2\pi MR^2}{5T}$ (d) $\frac{4\pi MR^2}{5T}$

Q.10 If the earth is a point mass of 6×10^{24}kg revolving around the sun at a distance of 1.5×10^8 km and in time T = 3.14 $\times10^7$s. then the angular momentum of the earth around the sun is

(a) 1.2×10^{18} kg m^2/s (b) 1.8×10^{29} kg m^2/s
(c) 1.5×10^{37} kg m^2/s (d) 2.7×10^{40} kg m^2/s

Q.11 An automobile engine develops 100 kW when rotating at a speed of 1800 rev/min. What torque does it deliver

(a) 350 N-m (b) 440 N-m
(c) 531 N-m (d) 628 N-m

Q.12 A constant torque acting on a uniform circular wheel changes its angular momentum from A_0 to $4A_0$ in 4 seconds. The magnitude of this torque is

(a) $\frac{3A_0}{4}$ (b) A_0
(c) $4A_0$ (d) $12A_0$

Q.13 A wheel having moment of inertia 2 kg -m^2 about its vertical axis, rotates at the rate of 60 rpm about this axis. The torque which can stop the wheel's rotation in 1 minute would be

(a) $\frac{2\pi}{15}$Nm (b) $\frac{\pi}{12}$Nm
(c) $\frac{\pi}{15}$Nm (d) $\frac{\pi}{18}$Nm

Q.14 Find the torque of a force $\vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$ acting at the point $\vec{r} = 7\hat{i} + 3\hat{j} + \hat{k}$

(a) $14\hat{i} - 38\hat{j} + 16\hat{k}$ (b) $4\hat{i} + 4\hat{j} + 6\hat{k}$
(c) $-14\hat{i} + 38\hat{j} - 16\hat{k}$ (d) $-21\hat{i} + 3\hat{j} + 5\hat{k}$

Q.15 A constant torque of 1000 N -m, turns a wheel of moment of inertia 200 kg -m^2 about an axis passing through the centre. Angular velocity of the wheel after 3 s will be

(a) 15 rad/s (b) 10 rad/s (c) 5 rad/s (d) 1 rad/s

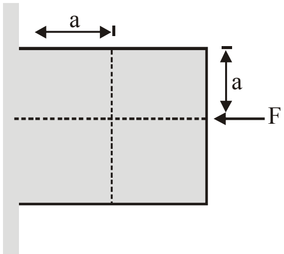
Q.16 A torque of 30 N-m is applied on a 5 kg wheel whose moment of inertia is 2kg-m^2 for 10 sec. The angle covered by the wheel in 10 sec will be

(a) 750 rad (b) 1500 rad
(c) 3000 rad (d) 6000 rad

RESPONSE GRID					
	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ
	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ			

Space for Rough Work

Q.17 A horizontal force F is applied such that the block remains stationary, then which of the following statement is false



(a) f = mg [where f is the friction force]
(b) F = N [where N is the normal reaction]
(c) F will not produce torque
(d) N will not produce torque

Q.18 In a bicycle, the radius of rear wheel is twice the radius of front wheel. If r_F and r_r are the radius, v_F and v_r are speeds of top most points of wheel, then
(a) $v_r = 2\, v_F$ (b) $v_F = 2\, v_r$
(c) $v_F = v_r$ (d) $v_F > v_r$

Q.19 The wheel of a car is rotating at the rate of 1200 revolutions per minute. On pressing the accelerator for 10 seconds, it starts rotating at 4500 revolutions per minute. The angular acceleration of the wheel is
(a) 30 rad/sec^2 (b) 1880 degree/sec^2
(c) 40 rad/sec^2 (d) 1980 degree/sec^2

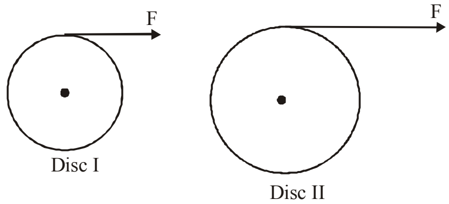
Q.20 A wheel rotates with a constant acceleration of 2.0 radian/sec^2. It the wheel starts from rest, the number of revolutions it makes in the first ten seconds will be approximately
(a) 8 (b) 16 (c) 24 (d) 32

DIRECTIONS (Q.21-Q.22) : In the following questions, more than one of the answers given are correct. Select the correct answers and mark it according to the following codes:

Codes :
(a) 1, 2 and 3 are correct **(b)** 1 and 2 are correct
(c) 2 and 4 are correct **(d)** 1 and 3 are correct

Q.21 A child is standing with folded hands at the centre of a platform rotating about its central axis. The kinetic energy of the system is K. The child now stretches his arms so that the moment of inertia of the system doubles. The kinetic energy of the system now is
(1) less than 2K (2) equal to K/2
(3) more thanK/4 (4) equal to 4K

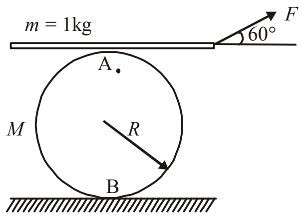
Q.22 Two uniforms discs of equal mass but unequal radii are mounted on fixed horizontal axiles. Light strings are wrapped on each of the discs. The strings are pulled by constant equal forces F for same amount of time as shown in the figure.



Angular momenta of discs are L_1 and L_2 and their kinetic energies are K_1 and K_2. Which of the following statements true –
(1) $L_1 = L_2$ (2) $L_1 < L_2$ (3) $K_1 > K_2$ (4) $K_1 = K_2$

DIRECTIONS (Q.23-Q.25) : Read the passage given below and answer the questions that follows :

Consider a cylinder of mass $M = 1$kg and radius $R = 1$ m lying on a rough horizontal plane. It has a plank lying on its top as shown in the figure.



A force F = 55 N is applied on the plank such that the plank moves and causes the cylinder to roll. The plank always remains horizontal. There is no slipping at any point of contact.

RESPONSE GRID	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ
	22. ⓐⓑⓒⓓ				

Space for Rough Work

Q.23 Calculate the acceleration of cylinder.

(a) 20 m/s^2 (b) 10 m/s^2

(c) 5 m/s^2 (d) None of these

Q.24 Find the value of frictional force at *A*

(a) 7.5 N (b) 5.0 N

(c) 2.5 N (d) None of these

Q.25 Find the value of frictional force at *B*

(a) 7.5 N (b) 5.0 N

(c) 2.5 N (d) None of these

DIRECTIONS (Q.26-Q.28) : Each of these questions contains two statements: Statement-1 (Assertion) and Statement-2 (Reason). Each of these questions has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(c) Statement -1 is False, Statement-2 is True.

(d) Statement -1 is True, Statement-2 is False.

Q.26 Statement -1: Torque is equal to rate of change of angular momentum.

Statement -2: Angular momentum depends on moment of inertia and angular velocity.

Q.27 Statement -1: Torque due to force is maximum when angle between $\vec{r}$ and $\vec{F}$ is 90°.

Statement -2: The unit of torque is newton- meter.

Q.28 Statement -1: It is harder to open and shut the door if we apply force near the hinge.

Statement -2: Torque is maximum at hinge of the door.

RESPONSE GRID	
	23. ⓐⓑⓒⓓ 24. ⓐⓑⓒⓓ 25. ⓐⓑⓒⓓ 26. ⓐⓑⓒⓓ 27. ⓐⓑⓒⓓ
	28. ⓐⓑⓒⓓ

DAILY PRACTICE PROBLEM SHEET 15 - PHYSICS			
Total Questions	28	Total Marks	112
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	28	Qualifying Score	44
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

Space for Rough Work

DAILY PRACTICE PROBLEMS

PHYSICS SOLUTIONS 15

1. (a) Initial angular momentum of ring. $L = I\omega = Mr^2\omega$
Final angular momentum of ring and four particles
system $Mr^2\omega = \left(Mr^2 + 4mr^2\right)\omega' = \frac{M\omega}{M+4m}$

2. (b) The angular momontum of a system of particles is con served when no external torque acts on the system.

3. (c) Rotational kinetic energy $E\frac{L^2}{2I}$ $\therefore L = \sqrt{2EI}$

$$\Rightarrow \frac{L_A}{L_B} = \sqrt{\frac{E_A}{E_B} \times \frac{I_A}{I_B}} = \sqrt{100 \times \frac{1}{4}} = 5$$

4. (c) Angular momentum $L = I\omega$ constant
$\therefore$ I increases and ω decreases

5. (c) Conservation of angular momentum
$I_1\omega_1 + I_2\omega_2 = (I_1\omega_1 + I_2)_\omega$

Angular velocity of system $\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$

$\therefore$ Rotational kinetic energy $= \frac{1}{2}\left(I_1 + I_2\right)\omega^2$

$$= \frac{1}{2}\left(I_1 + I_2\right)\left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2 = \frac{\left(I_1\omega_1 + I_2\omega_2\right)^2}{2\left(I_1 + I_2\right)}$$

6. (d) Kinetic energy $E = \frac{1}{2}L\omega = \frac{1}{2}L \times 2\pi n$

$$\therefore E \propto L \times n \Rightarrow \frac{L_2}{L_1} = \frac{E_2}{E_1} \times \frac{n_1}{n_2}$$

$$\frac{L_2}{L_1} = \left[\frac{E_1/2}{E_1}\right] \times \left[\frac{n_1}{2n_1}\right] \Rightarrow L_2 = \frac{L_2}{4} = \frac{L}{4}$$

7. (b) $E = \frac{L^2}{2I}$. If boy stretches hgis arm then moment of inertia increases and accordingly kinetic energy of the system decreases because $L =$ constant and $E \infty \frac{1}{I}$

8. (c)According to conservation of angular momentum

$$\therefore I_1\omega_1 = I_2\omega_2 \Rightarrow I_1\omega = \left(I_1 + I_2\right)\omega_2 \Rightarrow \omega_2 = \frac{I_1\omega}{I_1 + I_2}$$

9. (a) $L = \sqrt{2EI} = \sqrt{2 \times 10 \times 8 \times 10^{-7}} = 4 \times 10^{-3}\,\text{kg}\,\text{m}^2/\text{s}$

10. (d) Angular momentum, of earth about its axis of rotation,

$$L = I\omega = \frac{2}{5}MR^2 \times \frac{2\pi}{T} = \frac{4\pi MR^2}{5T}$$

11. (d) Angular momentum, $L = mvr = m\omega r^2 = m \times \frac{2\pi}{T} \times r^2$

$$= \frac{2 \times 3.14 \times 6 \times 10^{24} \times \left(1.5 \times 10^{11}\right)^2}{3.14 \times 10^7} = 2.7 \times 10^{40}\,\text{kg} - \text{m}^2/\text{s}$$

12. (c) $\omega = 2\pi n = \frac{2\pi \times 1800}{60} = 60\pi$ rad/s

$P = \tau \times \omega$

$$\Rightarrow \tau = \frac{P}{\omega} = \frac{100 \times 10^3}{60\pi} = 531\,N - m$$

13. (a) $\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{L_2 - L_1}{\Delta t} = \frac{4A_0 - A_0}{4} = \frac{3A_0}{4}$

14. (c) $\alpha = \frac{2\pi(n_2 - n_1)}{t} = \frac{2\pi\left(0 - \frac{60}{60}\right)}{60}$

$$= \frac{-2\pi}{60} = \frac{-\pi}{30}\ \text{rad/sec}^2$$
$\therefore \tau = I\alpha$
$$= \frac{2 \times \pi}{30} = \frac{\pi}{15} N - m$$

15. (a) $\vec{\tau} = \vec{r} \times \vec{f} = (7\hat{i} + 3\hat{j} + \hat{k}) \times (-3\hat{i} + \hat{j} + 5\hat{k})$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix}$$

$$= \hat{i}(15 - 1) - \hat{j}(35 + 3) + \hat{k}(7 + 9)$$
$$= 14\hat{i} - 38\hat{j} + 16\hat{k}$$

16. (a) $\alpha = \frac{\tau}{I} = \frac{1000}{200} = 5\ \text{rad/sec}^2$

From $\omega = \omega_0 + \alpha t = 0 + 5 \times 3 = 15$ rad/s

17. (a) $\alpha = \frac{\tau}{I} = \frac{30}{2} = 15\ \text{rad/s}^2$

$$\because \theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

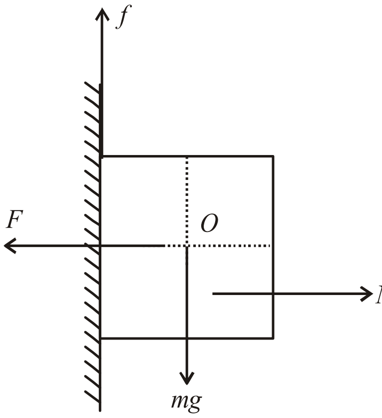
$$= 0 + \frac{1}{2} \times (15) \times (10)^2$$

$= 750$ rad

18. (d) As the block remains stationary therefore For translatory equilibrium

$\sum F_x = 0 \therefore F = N$

and $\sum F_y = 0 \therefore f = mg$



For rotational equilibrium $\sum \tau = 0$

By taking the torque of different forces about point 0

$\vec{\tau_F} + \vec{\tau_f} + \vec{\tau_N} + \vec{\tau_{mg}} = 0$

As F and mg passing through point O

$\therefore \vec{\tau_f} + \vec{\tau_N} = 0$

As $\tau_f \neq 0 \therefore \tau_N \neq 0$

and torque by friction and normal reaction will be in opposite direction.

19. (c) The velocity of the top point of the wheel is twice that of centre of mass and the speed of centre of mass is same for both the wheels (Angular speeds are different).

20. (d) $\alpha = \dfrac{2\pi(n_2 - n_1)}{t} = \dfrac{2\pi\left(\dfrac{4500-1200}{60}\right)}{10}$ rad/s^2

$$= \frac{2\pi \dfrac{3300}{60}}{10} \times \frac{360}{2\pi} \frac{\text{degree}}{s^2}$$

$\alpha = 1980$ degree/s^2

21. (b) $\theta = \omega_0 t + \dfrac{1}{2}\alpha t^2$

$\Rightarrow \theta = 100$ rad

$\therefore$ Number of revolution $= \dfrac{100}{2\pi} = 16$ (approx.)

22. (a) As mechanical contact is not made, total angular momentum remains constant.

$\therefore I\omega_0 =$ constant

Differentiating both sides,

$\Delta (I\omega_0) = 0$

$\Rightarrow I\,\Delta\omega_0 + \omega_0 \Delta I = 0$

$\Rightarrow \dfrac{\Delta\omega}{\omega} + \dfrac{\Delta I}{I} = 0 \Rightarrow \dfrac{\Delta I}{I} = -\dfrac{\Delta\omega_0}{\omega_0}$

Also, $\dfrac{\Delta\omega_0}{\omega_0} = -\dfrac{\Delta I}{I}$

$= -\dfrac{2\Delta R}{R} \left(\because \dfrac{\Delta I}{I} = \dfrac{2\Delta R}{R}\right) = -2\,\alpha\,\Delta T$

23. **(a)** $E = \dfrac{L^2}{2I} = K$ (given) $\therefore K \propto \dfrac{1}{I}$ (If L = constant)

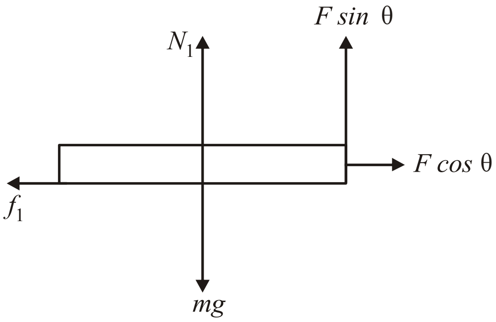
When child stretches his arms the moment of inertia of system get doubled so kinetic energy will becomes half i.e. K/2.

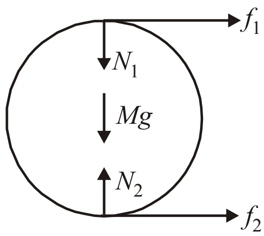
24. **(c).** Angular impulse = change in angular momentum : Frt = L $\Rightarrow L_1 < L_2$

$K = \dfrac{L^2}{2I} \Rightarrow K_1 = K_2$

25. (b); 26. (a); 27. (c)

Drawing the F.B. D of the plank and the cylinder.





Equations of motion are

$F\cos\theta - f_1 = ma$ ....(1)

$F\sin\theta + N_1 = mg$ ....(2)

$f_1 + f_2 = MA$ .....(3)

$f_1 R - f_2 R = I\alpha$ .....(4)

$A = R\alpha$ .....(5)

$$a = \frac{4F\cos\theta}{3M + 8m} = \frac{4 \times 55 \times \frac{1}{2}}{[(3\times1) + (8\times1)]} = 10 \text{ m/s}^2$$

$$f_1 = \frac{3MF\cos\theta}{3M+8m} = \frac{3\times1\times55\times\frac{1}{2}}{3\times1+8\times1} = 7.5\ N$$

and $$f_2 = \frac{MF\cos\theta}{3M+8m} = \frac{1\times55\times\frac{1}{2}}{3\times1+8\times1} = 2.5\ N$$

28. **(b)** $\vec{\tau} = \frac{d\vec{L}}{dt}$ and $L = I\omega$

29. **(b)** $\tau = rF\sin\theta$. If $\theta = 90°$ then $\tau_{max} = rF$

Unit of torque is N-m.

30. **(d)** Torque = Force × perpendicular distance of the line of action of force from the axis of rotation (d).

Hence for a given applied force, torque or true tendency of rotation will be high for large value of d. If distance d is smaller, then greater force is required to cause the same torque, hence it is harder to open or shut down the door by applying a force near the hinge.